Homework

1. Find the determinants of the matrices $det(A)$. Find out which of these matrices are invertible, and use the determinant to find out for which values of the constant the given matrix A is invertible.

(1) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$ (2)$A = \begin{pmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{pmatrix}$ (3)$A = \begin{pmatrix} 3 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{pmatrix}$

(4) $A = \begin{pmatrix} k & 3 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 4 \end{pmatrix}$ (5) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & k & -1 \\ 1 & k^2 & 1 \end{pmatrix}$ (6) $A = \begin{pmatrix} cosk & 1 & -sink \\ 0 & 2 & 0 \\ sink & 0 & cosk \end{pmatrix}$

(7)$A = \begin{pmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{pmatrix}$ (8) $A = \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ d & 0 & 0 & 0 \end{pmatrix}$ (9) $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{pmatrix}$

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2. Use the determinant to find out for which values of the constant λ the matrix $A - \lambda I_n$ fails to be invertible. (1) $A = \begin{pmatrix} 5 & 7 & 11 \\ 0 & 3 & 13 \\ 0 & 0 & 2 \end{pmatrix}$ (2) $A = \begin{pmatrix} 4 & 2 & 0 \\ 4 & 6 & 0 \\ 5 & 2 & 3 \end{pmatrix}$

3.Find nonzero numbers a, b, c, d, e, f, g, h such that the matrix $A = \begin{pmatrix} a & b & c \\ d & k & e \\ f & g & h \end{pmatrix}$ is invertible for all real numbers k or explain why no such matrix exists.

4.(1)Let A, B be 3×3 matrices with $det(A) = -3, det(B) = 4.$

Compute: $det(AB)$, $det(5\text{A}), det(B^{\text{T}})$, $det(A^{-1})$, $det(A^3)$.

(2)Let A, B be 4×4 matrices with $det(A) = -3, det(B) = -1.$

Compute: $det(AB)$,$det(B^5), det(2\text{A}), det(A^{\text{T}}BA), det(B^{-1}AB)$.

5. True or false: Let A, B be $n \times n$ matrices

(1) $det(A + B) = det(A) + det(B)$ (2) $det(I - AB) = det(AB - I)$