# Homework

1. Determine the value of the parameter m for which the system has a unique solution and describe the solution in terms of m.

$$\begin{cases} mx_1 + x_2 + x_3 = 1 \\ x_1 + mx_2 + x_3 = m \\ x_1 + x_2 + mx_3 = m^2 \end{cases}$$

2. Find the adjugate and inverse of $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$.

3. Find the classical adjoint of $A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

4. Let $A$ and $B$ be two invertible $n \times n$ matrices.

(1)What is $\mathbf{adj}(\mathbf{adj}A)$? (2)What is the relationship between $\mathbf{adj}A$ and $\mathbf{adj}(A^{-1})$?

(3) What is the relationship between $\mathbf{adj}A, \mathbf{adj}\mathrm{B}$ and $\mathbf{adj}(AB)$?

# Homework(Optional)

1. In an economics text[11] we find the following system:

$$\begin{bmatrix} -R_1 & R_1 & -(1-\alpha) \\ \alpha & 1-\alpha & -(1-\alpha)^2 \\ R_2 & -R_2 & \dfrac{-(1-\alpha)^2}{\alpha} \end{bmatrix} \begin{bmatrix} dx_1 \\ dy_1 \\ dp \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -R_2 de_2 \end{bmatrix}.$$

Solve for $dx_1$, $dy_1$, and $dp$. In your answer, you may refer to the determinant of the coefficient matrix as $D$. (You need not compute $D$.) The quantities $R_1$, $R_2$, and $D$ are positive, and $\alpha$ is between zero and one. If $de_2$ is positive, what can you say about the signs of $dy_1$ and $dp$?

2. Show that an $n \times n$ matrix $A$ has at least one nonzero minor if (and only if) $\operatorname{rank}(A) \geq n-1$.

3. Even if an $n \times n$ matrix $A$ fails to be invertible, we can define the adjoint $\operatorname{adj}(A)$ as in Theorem 6.3.9. The $ij$th entry of $\operatorname{adj}(A)$ is $(-1)^{i+j}\det(A_{ji})$. For which $n \times n$ matrices $A$ is $\operatorname{adj}(A) = 0$? Give your answer in terms of the rank of $A$. See Exercise 2.

4. Show that $A(\operatorname{adj}A) = 0 = (\operatorname{adj}A)A$ for all noninvertible $n \times n$ matrices $A$. See Exercise 3.

5. If $A$ is an $n \times n$ matrix of rank $n-1$, what is the rank of $\operatorname{adj}(A)$? See Exercise 3. and 4.