Homework

1. Let $V = Span\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_m}\}$. Find the coordinates of $\vec{x}$ with respect to the basis $\mathcal{B} = \{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_m}\}$ of V and write the coordinate vector $[\vec{x}]_{\mathcal{B}}$.

(a) $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$ (b) $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$; $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \end{bmatrix}$

2. Consider the basis $\mathfrak{B}$ of $\mathbb{R}^2$ consisting of the vectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$. We are told that $[\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$ for a certain vector $\vec{x}$ in $\mathbb{R}^2$. Find $\vec{x}$.

3. Find a basis $\mathfrak{B}$ of $\mathbb{R}^2$ such that

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{\mathfrak{B}} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{\mathfrak{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

4. Consider the plane $2x_1 - 3x_2 + 4x_3 = 0$ with basis $\mathfrak{B}$ consisting of vectors $\begin{bmatrix} 8 \\ 4 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$. If $[\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, find $\vec{x}$.

5. Consider the plane $2x_1 - 3x_2 + 4x_3 = 0$. Find a basis $\mathfrak{B}$ of this plane such that $[\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ for $\vec{x} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$.

Homework

6. Let $\mathcal{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be bases for a vector space V, and suppose $\mathbf{a}_1 = 4\mathbf{b}_1 - \mathbf{b}_2$, $\mathbf{a}_2 = -\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$, and $\mathbf{a}_3 = \mathbf{b}_2 - 2\mathbf{b}_3$.

   a. Find the change-of-coordinates matrix from $\mathcal{A}$ to $\mathcal{B}$.

   b. Find $[\mathbf{x}]_{\mathcal{B}}$ for $\mathbf{x} = 3\mathbf{a}_1 + 4\mathbf{a}_2 + \mathbf{a}_3$.

7. Let $\overrightarrow{b_1} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}, \overrightarrow{b_2} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}, \overrightarrow{c_1} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}, \overrightarrow{c_2} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, and consider two bases $\mathcal{B} = \{\overrightarrow{b_1}, \overrightarrow{b_2}\}$ and $C = \{\overrightarrow{c_1}, \overrightarrow{c_2}\}$ of R^2. Find the change-of-coordinates matrix from $\mathcal{B}$ to C and the change-of-coordinates matrix from C to $\mathcal{B}$.

8. Find the bases and dimensions for the four subspaces associated with A and B.
   (Hint: You can use the factorization to skip some elimination steps)

$$A = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 6 & 13 & 20 & 27 \\ 9 & 26 & 44 & 62 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 9 & 8 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$