Homework

Consider the transformations from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined in Exercises 1 through 3. Which of these transformations are linear?

1. $y_1 = 2x_2$
$y_2 = x_2 + 2$
$y_3 = 2x_2$

2. $y_1 = 2x_2$
$y_2 = 3x_3$
$y_3 = x_1$

3. $y_1 = x_2 - x_3$
$y_2 = x_1 x_3$
$y_3 = x_1 - x_2$

4. $T: R^2 \to R^2$ is a linear transformation that maps $\vec{u} = \binom{5}{2}$ into $\binom{2}{1}$ and maps $\vec{v} = \binom{1}{3}$ into $\binom{-1}{3}$. Use the fact that T is linear to find the images under T of $3\vec{u}, 2\vec{v}$ and $3\vec{u} + 2\vec{v}$.

5. Let $\overrightarrow{e_1} = \binom{1}{0}, \overrightarrow{e_2} = \binom{0}{1}$ and $T: R^2 \to R^2$ be a linear transformation satisfying

$T(\overrightarrow{e_1}) = \binom{2}{5}, T(\overrightarrow{e_2}) = \binom{-1}{6}$. Find the images of $\binom{5}{-3}, \binom{x_1}{x_2}$.

6. True or false: Let $T: R^n \to R^m$ be a linear transformation.

(1) If $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_k}\}$ is linearly independent in R^n, then $\{T(\overrightarrow{v_1}), T(\overrightarrow{v_2}), \cdots, T(\overrightarrow{v_k})\}$ is linearly independent in R^m.

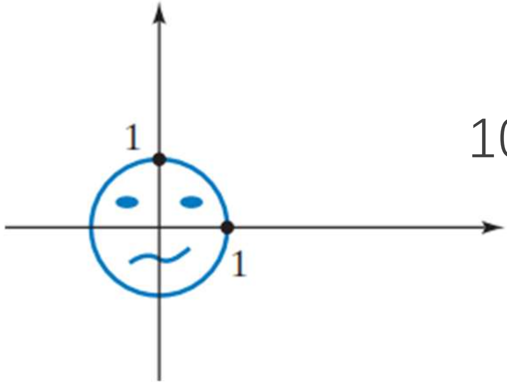
(2) If $\{T(\overrightarrow{v_1}), T(\overrightarrow{v_2}), \cdots, T(\overrightarrow{v_k})\}$ is linearly independent in R^m, then $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_k}\}$ is linearly independent in R^n.

Homework

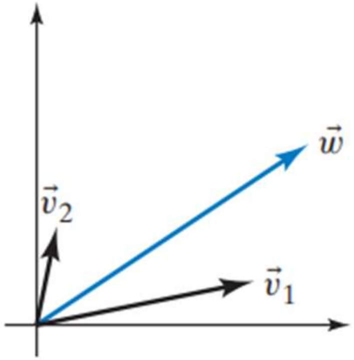
7. ***Consider the circular face in the accompanying figure. For each of the matrices A in Exercises* a *through* d *, draw a sketch showing the effect of the linear transformation $T(\vec{x}) = A\vec{x}$ on this face.***

a. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ d. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$



8. Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$. Let $\vec{v}_1$, $\vec{v}_2$, and $\vec{w}$ be three vectors in $\mathbb{R}^2$, as shown below. We are told that $T(\vec{v}_1) = \vec{v}_1$ and $T(\vec{v}_2) = 3\vec{v}_2$. On the same axes, sketch $T(\vec{w})$.



9. Find the inverse of the linear transformation

$$\begin{aligned} y_1 &= x_1 + 7x_2 \\ y_2 &= 3x_1 + 20x_2. \end{aligned}$$

10. Consider two linear transformations $\vec{y} = T(\vec{x})$ and $\vec{z} = L(\vec{y})$, where T goes from $\mathbb{R}^m$ to $\mathbb{R}^p$ and L goes from $\mathbb{R}^p$ to $\mathbb{R}^n$. Is the transformation $\vec{z} = L(T(\vec{x}))$ linear as well? [The transformation $\vec{z} = L(T(\vec{x}))$ is called the *composite* of T and L.]

11. Let T be a function from $\mathbb{R}^m$ to $\mathbb{R}^n$, and let L be a function from $\mathbb{R}^n$ to $\mathbb{R}^m$. Suppose that $L(T(\vec{x})) = \vec{x}$ for all $\vec{x}$ in $\mathbb{R}^m$ and $T(L(\vec{y})) = \vec{y}$ for all $\vec{y}$ in $\mathbb{R}^n$. If T is a linear transformation, show that L is linear as well. *Hint:* $\vec{v} + \vec{w} = T(L(\vec{v})) + T(L(\vec{w})) = T(L(\vec{v}) + L(\vec{w}))$ since T is linear. Now apply L on both sides.

Homework (Optional)

1. Give an example of a function $T\colon\ R^2 \to R$ such that $T(c\vec{x}) = cT(\vec{x})$ for all $\vec{x} \in R^2$ and $c \in R$ but T is not linear.

2. Let C be the set of complex numbers. Give an example of a function $T: C \to C$ such that $T(x+y) = T(x) + T(y)$ for all $x, y \in C$ but T is not linear.

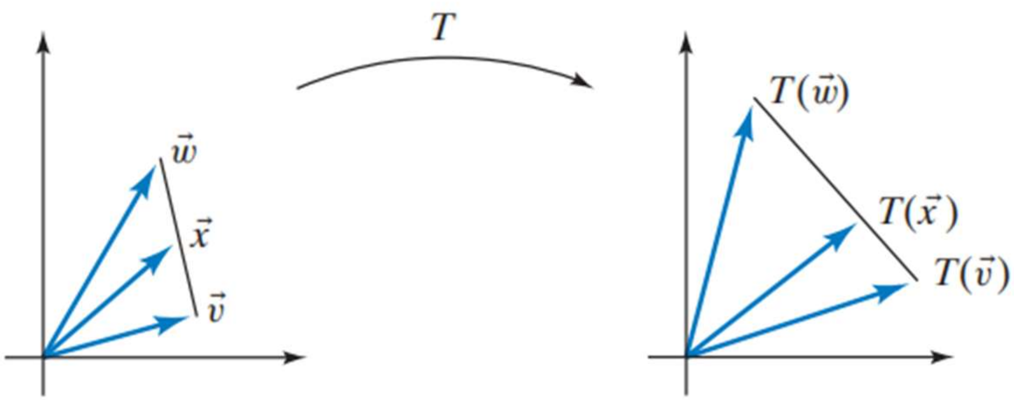
3. Which of the transformations are invertible?

(1) ***f from*** $\mathbb{R}$ to $\mathbb{R}$: $f(x) = x^3 - x$

(2) ***the (nonlinear) transformation from*** $\mathbb{R}^2$ ***to*** $\mathbb{R}^2$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1^3 + x_2 \end{bmatrix}$$

4. Consider a linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$. Suppose that $\vec{v}$ and $\vec{w}$ are two arbitrary vectors in $\mathbb{R}^2$ and that $\vec{x}$ is a third vector whose endpoint is on the line segment connecting the endpoints of $\vec{v}$ and $\vec{w}$. Is the endpoint of the vector $T(\vec{x})$ necessarily on the line segment connecting the endpoints of $T(\vec{v})$ and $T(\vec{w})$? Justify your answer.



Hint: We can write $\vec{x} = \vec{v} + k(\vec{w} - \vec{v})$, for some scalar k between 0 and 1.

We can summarize this exercise by saying that a linear transformation maps a line onto a line.