Homework

1. Let T is a linear transformation with standard matrix $A = \begin{pmatrix} -5 & 10 & -5 & 4 \\ 8 & 3 & -4 & 7 \\ 4 & -9 & 5 & -3 \\ -3 & -2 & 5 & 4 \end{pmatrix}$.

   Is T one-to-one? Does T map R^4 onto R^4?

2. Let T: $R^n \to R^m$ be a linear transformation.

   If T one-to-one, what can you say about m and n?

   If T map R^n onto R^m, what can you say about m and n?

3. Let V be the subspace of $\mathbb{R}^4$ defined by the equation

$$x_1 - x_2 + 2x_3 + 4x_4 = 0.$$

   Find a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^4$ such that $\ker(T) = \{\vec{0}\}$ and $\text{im}(T) = V$. Describe T by its matrix A.

4.(a)Give an example of a linear transformation whose image is the line spanned by $\begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix}$ in R^3.

(b) Give an example of a linear transformation whose kernel is the line spanned by $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ in R^3.

Homework

Image of a linear transformation

The image of a linear transformation $T(\vec{x}) = A\vec{x}$ is the span of the column vectors of A.[2] We denote the image of T by $\text{im}(T)$ or $\text{im}(A)$.

Kernel

The *kernel*[3] of a linear transformation $T(\vec{x}) = A\vec{x}$ from $\mathbb{R}^m$ to $\mathbb{R}^n$ consists of all zeros of the transformation, that is, the solutions of the equation $T(\vec{x}) = A\vec{x} = \vec{0}$. See Figure 10, where we show the kernel along with the image.

In other words, the kernel of T is the solution set of the linear system

$$A\vec{x} = \vec{0}.$$

We denote the kernel of T by $\ker(T)$ or $\ker(A)$.

5. Can you find a 3×3 matrix A such that $\text{im}(A) = \ker(A)$? Explain.

6. Give an example of a 4×5 matrix A with $\dim(\ker A) = 3$.

7. **a.** Consider a linear transformation T from $\mathbb{R}^5$ to $\mathbb{R}^3$. What are the possible values of $\dim(\ker T)$? Explain.

   b. Consider a linear transformation T from $\mathbb{R}^4$ to $\mathbb{R}^7$. What are the possible values of $\dim(\text{im}T)$? Explain.