Homework

1. Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. (a) Find the real eigenvalues (b) Find the complex eigenvalues

2. Check if $\vec{u} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of $A = \begin{pmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{pmatrix}$

3. Find the eigenvalues of A and find a basis for the corresponding eigenspaces.

Diagonalize the matrix if it's diagonalizable.

(1) $A = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$ (2) $A = \begin{pmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}$ (3) $A = \begin{pmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

4. Determine if the following matrices are diagonalizable.

(1) A is a 3×3 matrix with two eigenvalues. Each eigenspace is one-dimensional

(2) A is a 4×4 matrix with three eigenvalues. One eigenspace is one-dimensional and one of the other eigenspaces is two-dimensional

(3) A is a 7×7 matrix with three eigenvalues. One eigenspace is two-dimensional and one of the other eigenspaces is three-dimensional

Homework

5. Compute A^k (k is a positive integer). Let $A = \begin{pmatrix} a & 0 \\ 3(a-b) & b \end{pmatrix}$.

6. True or False

(1) Suppose A is invertible. Then λ is an eigenvalue of $A \Leftrightarrow \lambda^{-1}$ is an eigenvalue of A^{-1}

(2) λ is an eigenvalue of $A \Leftrightarrow \lambda$ is an eigenvalue of A^T

(3) Suppose k is a positive integer. If A is similar to B, then A^k is similar to B^k.

(4) If A, B are $n \times n$ matrices and A is invertible, then AB is similar to BA.

(5) If A is invertible, A is diagonalizable.

(6) If $\vec{v}$ is an eigenvector of both A and B, then $\vec{v}$ is necessarily an eigenvector of AB.